Nonparametric Interest Rate Cap Pricing and Implications for the "Unspanned Stochastic Volatility" Puzzle

Tao L Wu*

School of Management
SUNY at Buffalo

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Abstract

Asset prices depend on two elements: the dynamics of the state variables and the pricing kernel. Traditional term structure models differ in the factor dynamics. However, most of them imply a log-linear pricing kernel. We investigate empirically the role of factor dynamics and pricing kernel in pricing interest rate derivatives using a nonparametric approach. We find that the prices of interest rate caps are very sensitive to the specification of the factor dynamics, especially when they are close to expiration. In addition, nonlinear log-pricing kernels improve the pricing of long-maturity caps. Recent research document models that fit libor and swap rates but do not price derivatives well, leading to the so called "unspanned stochastic volatility puzzle". Additional factors seem to be needed to explain cap prices. However, the relative mispricing between interest rate caps and underlying libor and swap rates could also potentially be due to mis-specification of the parametric models used. Our paper provides evidence, from a nonparametric perspective, for the inability of diffusion-only type of models to price interest rate caps. Models with jumps such as that in Jarrow, Li, and Zhao (JF, forthcoming) is needed to explain the prices of libor options.

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* Email: twu@buffalo.edu.

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Introduction

Asset prices are governed by two major elements: the dynamics of state variables and the pricing kernel.[1] In this paper we study the factor dynamics of the short term interest rate, the nonlinearity of the pricing kernel, and their implications for out-of-sample pricing of interest rate derivatives. This is an interesting topic because factor dynamics, especially the volatility dynamics are particularly important for pricing option-like contingent claims. We find that the prices of interest rate caps are extremely sensitive to different factor dynamics, especially for short maturity at the money caps. In addition, nonlinear log pricing kernels price long maturity caps better than linear log pricing kernels in a model that closely captures the time-varying conditional non-normality in the factor dynamics of the short rate. Finally, significant pricing errors for caps remain even with the most successful nonparametric model estimated using libor and swap rates alone. This finding is consistent with recent research of parametric models that fit libor and swap rates but do not price derivatives well. Jaganathan, Kaplin and Sun (2003) show that three-factor CIR models produce large pricing errors for caps and swaptions. Collin-Dufresne and Goldstein (2002), Heidari and Wu (2003), present evidence for the "unspanned stochastic volatility" puzzle. They note that risk factors driving the caps and swaptions market are unspanned by factors underlying the libor and swaps market. They suspect additional factors are needed to explain cap prices.[2] Li and Zhao (2005) also report that although quadratic term structure models fit libor and swap rates well, they lead to poor hedging performance of caps, especially cap straddles. However, the relative mispricing between interest rate caps and underlying libor and swap rates could also be due to mis-specification of the parametric models used. The results obtained using a non-parametric approach in this paper further confirm the inability of diffusion-only type of models to price interest rate caps. Models with jumps such as that in Jarrow, Li, and Zhao (JF, forthcoming) is needed to explain the prices of libor options.

Most theoretical models of the term structure of interest rate imply a log linear pricing kernel but differ in the dynamics of the state variables. Among the equilibrium pricing models, for example, Vasicek (1977), Dothan (1978), and Cox, Ingersoll and Ross (1985) specifies the diffusion term to be a constant, a linear, and a square root function of the short rate, respectively. In addition, Duffie and Kan (1996) propose a general class of

[1]The pricing kernel is also known as the state price density, the state-price deflator, and the stochastic discount factor.

[2]Fan, Gupta, Ritchken (2003) show that improvement on hedging performance of swaptions by including additional factors are small, but do not address caps.

affine yield model[3], where the diffusion term is proportional to the square root of an affine function of the state-variables. Finally, Constantinides and Ingersoll (1984), and Ahn and Gao (1999) make the diffusion term proportional to the 1.5th power of the short rate. No-arbitrage term structure models, for example, Black, Derman and Toy (1990), Hull and White (1990), and Heath, Jarrow and Morton (1992), introduce time-varying parameters to match the observed volatility term structure. Among empirical studies, Ait-Sahalia (1996b) tests diffusion terms in the form of the square root of a quadratic function of the short rate; Boudoukh, Richardson, Stanton, and Whitelaw (1999) estimate nonparametrically the diffusion term as a function of the level and the slope of the yield curve. No concensus on the appropriate model for the factor dynamics has been reached.

No-arbitrage models allow enough free parameters to match the initial term structure of interest rates and in some cases, also the initial term structure of volatilities. However, they are really just reduced-form models in that they do not link the term structure of the interest rates to economic fundamentals, such as the production technology and preferences. Therefore, the danger exists for over-parameterizing the model, which may cause them to imply unrealistic time series properties of the state variables and to perform poorly for out of sample pricing and hedging. In fact, most no-arbitrage models need to be recalibrated to a different set of parameters every day.

Unlike no-arbitrage models, an equilibrium model is developed in a structural fashion and is embedded in an underlying equilibrium economy. The factor dynamics are estimated using time-series data. Unfortunately, these models often cannot match the cross-section of bond prices at any point in time.

In this paper, we follow an integrated approach developed in Brandt and Yaron (2003)[4]. Like equilibrium pricing models, it recovers the underlying factor dynamics using a very flexible semiparametric model. At the same time, like no-arbitrage models, it can match the cross-section of bond prices through non-parametric pricing kernels. Specifically, we first estimate the factor dynamics from historical time-series. Then we use the cross-section of zero-coupon bond prices to infer the pricing kernel. Finally, we combine the estimated factor dynamics and inferred pricing kernel to price interest caps out-of-sample. We explore what are the necessary features in the volatility structure and pricing kernel that lead to more accurate pricing.

We estimate a Gaussian VAR(1) model, a Gaussian VAR-GARCH(1,1) model, an un-

[3]See also Dai and Singleton (2000).

[4]Their paper focuses only on bond pricing, not on the pricing of derivatives.

conditional non-Gaussian VAR-GARCH(1,1) model, and a conditional non-Gaussian VAR-GARCH(1,1) model. In addition, we examine the relative impact of using linear, quadratic, and cubic log pricing kernels on the pricing of interest caps out of sample. The last model nests a number of standard term structure models as its special cases. As in Backus, Foresi, and Telmer (1998), we adopt a discrete time and continuous state variables framework. More details of this setting will be given in the theory section.

The paper proceeds as follows. In Section I, we review the literature to further motivate our approach. Section II describes the theoretical framework. Section III discusses the econometric methodology. In Section IV we apply the model to the LIBOR and swap markets. Section V describes the pricing of interest rate caps. In Section VI, we present the empirical results. Section VII concludes with a summary of results and suggestions for future research.

I. Previous Related Research

The first no-arbitrage model is developed by Ho and Lee (1986). Their model introduces a set of time-dependent parameters in the drift term to exactly fit the current term structure of interest rates. However, the Ho-Lee model does not allow for mean reversion. Moreover, it describes the volatility structure of the short rate with only one parameter, which is inconsistent with reality. Black, Derman and Toy (1990) overcome this latter problem by allowing the volatility parameter to be time-varying in order to match the initial volatility term structure. However, mean-reversion still remains an unresolved issue in the Black-Derman-Toy model.[5] Hull and White (1990, 1993) approach no arbitrage models by adding time-dependent parameters to equilibrium models. The drawback is that this approach in general does not provide closed-form solutions, except for the extended Vasicek model. Heath, Jarrow and Morton (1992) model the forward rate curve and develop no arbitrage restrictions on the drift and diffusion terms of the forward rate processes under the equivalent martingale measure. The Heath-Jarrow-Morton model can be calibrated to match the initial interest rate and volatility term structure. Unfortunately, except for a few known cases of forward volatility functions, the short rate implied by the Heath-Jarrow-Morton approach is non-Markovian, which makes pricing by simulation a formidable task.

Despite its popularity among practitioners, no-arbitrage models have drawn consider-

[5] For certain volatility structures, the short rate can be mean-escaping rather than mean-reverting in the Black-Derman-Toy model.

able concerns from academics. The time-varying parameters in these models provide little economic insights. For example, Dybvig (1989) points out that no-arbitrage models are ad hoc in that they describe the dynamics of the yield curve by changes in the time-dependent parameters instead of changes in the underlying fundamental variables. The fact that no-arbitrage models need to be re-calibrated everyday suggests inappropriateness of the framework. Furthermore, Backus, Foresi, and Zin (1996) demonstrate that no-arbitrage models with mis-specified short rate dynamics can systematically misprice certain derivative assets. Specifically, they show that the Black-Derman-Toy model overprices call options on long bonds relative to those on short bonds when interest rates are mean-reverting. They conclude that time-dependent parameters must be used with close attention to fundamentals that drive bond prices.

Now let's turn to equilibrium term structure models. In a single factor world, assuming constant volatility of the short rate, Vasicek (1977) obtains a model where the short rate follows an Orstein-Uhlenbeck process. The short rate is Gaussian and can become negative with positive probability. Cox, Ingersoll and Ross (1985) assume a square root diffusion process for the state variable, production technology, and logarithmic utility function for the agents. They derive a single-factor model for the short rate with mean-reversion and diffusion term proportional to the square root of the short rate. Several multi-factor theoretical models have also been proposed. For example, Brennan and Schwartz (1989), Longstaff and Schwartz (1992) are among two-factor models with closed-form solutions for bond prices. Affine class models, where yields are affine functions of the interest rate, are popular among multi-factor equilibrium models for their analytical tractability and ease of implementation. These models provide closed-form solutions for transition and marginal densities of the short rate, as well as bond prices. Duffie and Kan (1996)[6] propose a very general class of affine model that nests Vasicek and CIR models. Affine models typically have an affine drift and square root diffusion, which have been called into question among empirical studies recently. In particular, Chan, Karolyi, Longstaff, and Sanders (1992) estimate a constant elasticity of volatility model (CEV model) using Euler approximation. They find that a diffusion term proportional to the 1.5-th power of the short rate reject all popular models. This result is further supported by results in Campbell, Lo, and MacKinlay (1996). Furthermore, Ait-Sahalia (1996a,b) also document significant non-linearity in both the drift and diffusion functions using a non-parametric procedure. Based on these observations, Ahn and Gao (1999) propose a single factor parametric model with a quadratic drift and a diffusion term proportional to the 1.5-th power of the short rate. This model allows for closed form solutions.

[6]See also Dai and Singleton (2000).

Boudoukh, Richardson, Stanton, and Whitelaw (1999) also find significant non-linearity in the volatility function. In particular, they observe that volatility of the short rate is increasing in the level of the short rate only when the term structure is sharply positively sloped.

To capture the potential non-linearity in both the drift and the diffusion functions, the factor dynamics is estimated using Gallant and Nychka (1987) Semi-Nonparametric (SNP) method. Intuitively, their SNP estimator models the state vector as a nonlinear (and potentially time-varying) transformation of a Gaussian VAR process with GARCH covariance matrix. Rather than picking a process arbitrarily, this model is generated from data. It maintains the flexibility of nonparametric method while at the same time seeks to shed some light on the possible candidate parametric forms to the drift and diffusion functions. Comparing with full blown non-parametric estimation, the SNP model avoids "the curse of dimensionality". Meanwhile, the approach we follow can match the cross-section of zero-coupon bond prices through a non-parametric pricing kernel. Since Boudoukh, Richardson, Stanton, and Whitelaw (1999) assume a fixed log linear pricing kernel, their model can be seen as an unconditional version of the model applied in this paper.

In the next two sections, we outline the model set up and estimation strategy used in this study.

II. Theory

We consider a frictionless economy with no arbitrage opportunities. The state of the economy is described by an L-dimensional state variable vector z_t. Harrison and Kreps (1979), among others, show that in such a setting, the absence of arbitrage ensures the existence of a positive pricing kernel M with the property that:

$$\mathrm{E}_t[M(z_t, z_{t+1})R_{i,t+1}] = 1 \tag{1}$$

where $R_{i,t+1}$ is the one-period return of asset i, and the expectation is taken with respect to all the information available to market participants at time t. Specifically, if $R_{i,t+1}$ is the return on the one period zero-coupon bond, we have

$$P_{1,t} = \mathrm{E}_t[M(z_t, z_{t+1})] \tag{2}$$

where $P_{1,t}$ is the time t price of a one period zero coupon bond. The price of a N-period zero coupon bond $P_{N,t}$ is given by

$$
\begin{aligned}
P_{N,t} &= \mathrm{E}_t[M(z_t, z_{t+1})P_{N-1,t+1}] \\
&= \mathrm{E}_t\Big[M(z_t, z_{t+1})\mathrm{E}_{t+1}[M(z_{t+1}, z_{t+2})P_{N-2,t+2}]\Big] \\
\cdots \\
&= \mathrm{E}_t\Big[\prod_{i=0}^{N-1} M(z_{t+i}, z_{t+i+1})\Big].
\end{aligned}
$$

The last equality follows by recursive substitution and the law of iterated expectations. Let $m = \log(M)$, we have

$$
P_{N,t} = \mathrm{E}_t\Big[\exp\Big\{\sum_{i=0}^{N-1} m(z_{t+j}, z_{t+j+1}; \theta)\Big\}\Big]. \tag{3}
$$

Like Backus, Foresi, and Telmer (1998), we adopt a discrete time and continuous state variables framework. Backus, Foresi, and Telmer translate continuous-time term structure models into the discrete time and continuous state variables setting. For example, the multifactor Vasicek model can be written in the following way. Independent and de-meaned state variables z_{it} follow

$$
z_{it+1} = \varphi_i z_{it} + \sigma_i \varepsilon_{it+1} \tag{4}
$$

with ε_{it} normally distributed with mean zero and variance one and independent across i and t. The log pricing kernel is given by

$$
m_{t+1} = \delta + \sum_i (\lambda_i^2/2 + z_{it} + \lambda_i \varepsilon_{it+1}) \tag{5}
$$

Substituting (1) into (2) and eliminate ε_{it+1}, we can express the log pricing kernel as linear functions of z_{it} and z_{it+1}. Both the CIR model and the Longstaff-Schwartz model can be translated in a similar fashion. More generally, any affine yield models can be written as the following:

$$
z_{t+1} = \Pi_0 + \Pi_1 z_t + V(z_t)^{1/2}\varepsilon_{t+1}, \tag{6}
$$

where $V(z_t)$ is a diagonal matrix with typical element $\alpha_i + \beta_i' z_t$. The log pricing kernel for this class of models is time-separable and linear in the state variables:

$$
m(z_t, z_{t+1}; \theta) = \theta_0 + \theta_1' z_t + \theta_2' z_{t+1}. \tag{7}
$$

Therefore, the approach we apply in this paper nests all the above models as special cases. In addition, Boudoukh, Richardson, Stanton, and Whitelaw (1999) show that stochastic volatility models, for example, the Longstaff-Schwartz model, can also be re-written in level and slope type of models in the affine-yield setting.

III. Empirical Methodology

In this section, we first specify the state variable vector z_t. Although single factor models allow for tractability, they have the undesirable implication that yields on bonds of different maturities are perfectly correlated instantaneously. In addition, numerous studies have suggested that multiple factors are needed to explain the behavior of bond yields. Stambaugh (1988), Litterman and Scheinkman (1991), Longstaff and Schwartz (1992), Pearson and Sun (1993), and Anderson and Lund (1997) all offer empirical evidence favoring multi-factor models to single factor models. Several multi-factor theoretical models have been proposed. For example, Brennan and Schwartz (1989) choose the short rate and the long rate, while Longstaff and Schwartz (1992) use the short rate and its volatility as the two factors. These choices were made for analytical tractability rather than empirical observations. Shafer and Schwartz (1984) choose the short rate and the slope of the yield curve as the two factors. This is confirmed by empirical evidence offered by Litterman and Scheinkman (1991), where they find the three common factors affecting bond yields are the level, the slope, and the curvature of the yield curve. In particular, the first two factors, the level and the slope can capture almost all (96 %) of the variations in bond yields. Boudoukh, Richardson, Stanton, and Whitelaw (1999) develop a two factor non-parametric model based on the level and the slope. They show how volatility is related to the slope and how their model is a generalized version of the Longstaff-Schwartz model,where volatility is the second factor. Based on these evidence, we propose using the short rate and the slope as our two factors in this paper. In particular, the level is measured by the 3-month zero-coupon bond yield, the slope is proxied by the spread of the ten-year over the one-year zero-coupon bond yields. Our model can be easily extended to incorporate a third factor, such as the curvature of the yield curve.

We first estimate the dynamics of z_t using a Gaussian VAR model and a Gaussian VAR-GARCH model. We then estimate the dynamics of z_t using a semi-nonparametric (SNP) formulation developed in Gallant and Nychka (1987). The method uses a Hermite polynomial series expansion to approximate the conditional density of a multivariate process. It is a nonlinear nonparametric model that nests the Gaussian VAR model, the semiparametric Gaussian VAR model, the Gaussian GARCH model, and the semiparametric GARCH model. The estimation uses maximum likelihood together with a model selection strategy that determines the appropriate degree of the polynomial. The estimator is consistent under reasonable regularity conditions. The SNP formulation can account for (potential) time varying conditional non-normality.

With the estimated dynamics of the state variables, we can simulate state variable vector

z K periods forward, conditioning on its current value z_t. K is the maximum maturity of the bonds in the cross-section. Repeating this n times gives us n paths of the possible future evolution of the state variable vector. Then we expand the log pricing kernel m using Hermite polynomials with parameters θ. Finally, we solve the following minimization problem over the θ:

$$\min_{\theta} \sum_{N \in \{1,2,\ldots K\}} \Big\{ -\frac{1}{N} \log \Big(\mathrm{E}_t \Big[\exp \Big\{ \sum_{i=0}^{N-1} m(z_{t+j}, z_{t+j+1}; \theta) \Big\} \Big] \Big) - (-\frac{1}{N} \log P_{N,t}) \Big\}^2. \quad (8)$$

In fact, we are minimizing the sum of squared differences between theoretical yields and observed yields. The summation is taken over all the zero-coupon bonds in the cross-section with prices $P_{N,t}$ respectively. The conditional expectation is evaluated by averaging the expression inside the expectation operator along the sample paths of future z_t's generated through simulation. Expanding $\log(M)$ instead of M ensures the positivity of the pricing kernel. To reduce the number of parameters required for the expansion, we assume time separability of the log pricing kernel:

$$m(z_t, z_{t+1}; \theta) = m_0 + m_1(z_t; \theta_1) + m_2(z_{t+1}; \theta_2), \quad (9)$$

where $m_1 \neq m_2$ and $\theta_1 \neq \theta_2$. This is an assumption satisfied by most term structure models, for example, the affine yield models. With time-separability, a second-order expansion with two state variables involves 12 parameters and a third-order expansion involves 20 parameters.[7]

IV. Estimation using the LIBOR and Swap Data

The LIBOR (London Inter-Bank Offer Rate)-swap yield curve is one of the most important objects in the fixed-income markets. The pricing of many fixed-income derivatives, such as swaps, interest caps, floors, collars, and swaptions are based on the LIBOR rates by convention. A swap is private agreement between two parties A and B. In a plain vanilla swap, Party A agrees to pay party B a fixed interest rate on a notional amount over the life of the swap. In exchange, Party B agrees to pay party A an amount based on a floating rate on the same notional over the same periods. Typically, the floating rate is the 3-month LIBOR. The fixed rate, known as the swap rate, is set such that the value of the swap contract is zero to both parties at the time of initiation.

[7] We expand the log pricing kernel m using complete (instead of tensor product) Hermite polynomials.

A. Description of the LIBOR and Swap Data

U.S. dollar LIBOR and swap rates are obtained from Data Stream. The time period is from April 7, 1987 to June 22, 1999. The LIBOR rates are for 3- and 6- month maturities. The swap rates are for maturities 1-, 2-, 3-, 5-, 7- and 10-years. All data are of weekly frequency, and there are a total of 638 observations.

To describe the behavior of the LIBOR and swap rates during the sample period, we graph the 3-month LIBOR rate, the 10-year swap rate, and the spread between the 10-year swap rate and the 1-year swap rate in Figure 1.

Casual observation show that rates fall in the first half of the sample period, reach their low around June 1993, and start increasing until early 1995. After that, they stay within a tight range for the remainder of the sample. The 10-year swap rate is generally higher than the 3-month LIBOR rate. In addition, the yield curve is on average positively sloped, with the exception of a short period between January and June 1989 and a few weeks in 1998, when the spread between the 10-year and 1-year swap rates falls below zero. The slope is negatively correlated with the short rate at a correlation of -0.625.

Table 1 reports the sample mean and standard error for the LIBOR and swap rates. The average 3-month LIBOR rate in our sample is 5.95% with a standard deviation of 1.78%. The average 10-year swap rate in the sample period is 7.75% with a standard deviation of 1.44%. The rates are on average increasing with maturity, while the level of the shorter maturity rates are more volatile than the longer maturity rates.

Table 2 reports the sample mean and standard error for the weekly changes in the LIBOR and swap rates. The average weekly change to the 3-month LIBOR is -0.22 basis points while the average weekly change to the 10-year swap rate is -0.28 basis points. The negative mean changes indicate that on average, rates fell over the past twelve years. Although the 3-month LIBOR is more volatile in level, it is less volatile in weekly changes than the 10-year swap rates. In fact, the short maturity swap rates are the most volatile in weekly changes.

B. Constructing Zero-coupon Yield Curve

It is conventional among interest rate derivative traders to use libor and swap rates to solve for the zero yield curve, and then use it to discount cash-flows. We follow this market convention to bootstrap the zero-coupon yield curve from the short maturity LIBOR rates and the swap rates. Specifically, the zero-coupon yield curve is constructed such that the swap contracts are worth zero at the time of initiation. Linear interpolation is used where necessary.

In constructing the zero-coupon yields this way, we assume that the default risk embedded in short maturity LIBOR and generic swap contracts is minimal. We only use the short end of the LIBOR curve as the short end of zero-coupon yield curve. The remaining part of the yield curve are constructed using generic swap rates that apply to top quality counter parties rated AA or better. Collin-Dufresne and Solnik (1998) point out that the default risk in these generic swap contracts are very small for two reasons. First, there is no exchange of principal in a swap transaction. A counterparty's potential loss is the interest differential (in the case of fixed versus floating swap), and only if the swap is in-the-money for the non-defaulting counterparty. Second, the existence of posting collaterals and mark-to-market requirements, as well as other contractual provisions further eliminate credit risks in swap contracts.

The zero-coupon yield curve constructed using swap rates is different from the one constructed using LIBOR par bond yields. LIBOR bonds are coupon bonds issued by top quality firms with ratings AA or better. They carry much higher default risks than swaps because swaps reflect "refreshed credit quality" of LIBOR counterparties, which means the counter-parties maintain the same credit rating over time. However, LIBOR par bond yields reflect the potential downgrade of the issuer.[8]

On the other hand, yields on Treasury bills are not suitable for our purpose. In fact, Grinblatt (1995) argues that Treasury securities have a liquidity advantage over privately-issued instruments because Treasury securities are ideal instruments for hedging interest rate risks. Therefore, Treasury securities provide a convenience yield in addition to coupon payments and price appreciation.

In an empirical study, Duffee (1996) concludes that when calibrating default-free term structures, short maturity Eurodollars (LIBOR) should be used instead of Treasury bill yields because the idiosyncratic variation in Treasury bill yields make them poor proxies for the instantaneous default-free interest rate. The first type of variation is the movement of individual short maturity Treasury bill yields linked to bill supply that are unrelated to any other instruments. He suggests that this reflects increased market segmentation. The second type of variation is the common movement in Treasury bill yields that are unrelated to other instruments such as long maturity Treasury notes and short maturity privately-

[8]See Collin-Dufresne and Solnik (1998), Litzenberger (1992) and Solnik (1990) for detailed discussions.

issued instruments. As a result, he finds that since the early 1980's, the correlation between the monthly changes in Treasury bill yields with contemporaneous changes in yields of other instruments have significantly declined. Furthermore, he points out that no-arbitrage term structure models are not models for Treasury yields. The short rate in these models is the risk-free rate where agents can lend and borrow without credit risks, and agents cannot borrow at the same rate as the U.S. government. Though no-arbitrage models are not models for yields on private-issued securities either, the minimal default risk in short maturity securities issued by firms rated AA or better makes LIBOR closer to risk-free rate than Treasury bill yields are.

V. Interest Rate Cap Valuation

In the previous section we have estimated our models using the LIBOR and swaps rates. Now we are ready to apply the models to pricing interest rate derivatives. Specifically, we will use prices of interest caps written on the 3-month LIBOR to evaluate the performance of different models. Interest caps are insurance policies against rising interest. They are provided by financial institutions in over-the-counter market. A cap is typically described by the cap rate X, settlement period τ, and maturity T. Assuming interest payment are made at times τ, 2τ,... $n\tau$=T, on a notional amount of \$1, then the buyer of the cap at time $(k+1)\tau$ receive

$$\tau(R_k - X)^+ \tag{10}$$

where R_k is the level at time $k\tau$ of the rate being capped. Each individual call option like this is called a caplet. A cap is just a portfolio of caplets.

A. Black's Model for Cap Pricing

It is a market convention to quote the price of a cap in terms of Black's volatility, similar to equity option traders quoting implied volatilities instead of prices. Black's model assumes a lognormal process for the LIBOR rate. The price of a cap is then given by (Hull 1997)

$$C = \tau \sum_{k=1}^{n} \exp\{-r(k+1)\tau\}[F_k N(d_{1k}) - XN(d_{2k})] \tag{11}$$

where

$$d_{1k} = \frac{\ln(F_k/X) + \sigma_k^2 k\tau/2}{\sigma_k\sqrt{k\tau}} \tag{12}$$

$$d_{2k} = d_{1k} - \sigma_k \sqrt{k\tau} \tag{13}$$

and F_k is the forward LIBOR rate between date $k\tau$ and $(k+1)\tau$. Black's volatility of F_k σ_k is assumed to be constant. The price of the cap is then the summation of individual caplets. There are two internal inconsistencies in this model. First, the forward rate is assumed to be stochastic, while the risk-free rate used for discounting is assumed to be constant. Second, the assumption that forward rate volatility is constant is an approximation. In reality, forward rate volatility decreases with maturity as long maturity forward rates are less affected by the changes in the current level of intereset rates. Despite these two problems, Black's model is used as a tool to transform cap implied volatilities to prices.

B. Description of the Caps Data

We obtain from Data Stream weekly cap data from March 18, 1997 to June 22, 1999. The prices are for at the money caps on 3-month LIBOR, quoted in terms of Black's volatilities. For each observation, there are seven different maturity contracts, 1-, 2-, 3-, 4-, 5-, 7-, and 10-years. We translate the volatility quotes into corresponding prices in basis points (one percent of a cent on a notional of $1).

Table 3 presents the mean and standard deviation of Black's volatilities for the caps. We also graph them in Figure 2 and Figure 3. The mean volatilities of different maturity caps exhibit a slightly hump-shaped structure, with long maturity Black's volatility higher than those of the 1 year caps. However, the prices of short maturity caps are more volatile in terms of quoted volatilities.

C. Cap Pricing using Pricing Kernel

By definition of the pricing kernel, the price of the k-th caplet is given by

$$C_k = \mathrm{E}_t\Big[\exp\{\sum_{j=0}^{k-1} m(z_{t+j}, z_{t+j+1}; \theta)\}(R_k - X)^+\tau\Big]. \tag{14}$$

This can be computed by averaging the expression inside the expectation operator along the n simulated paths of state variables:

$$C_k = \frac{1}{n}\sum_{i=1}^{n} \exp\{\sum_{j=0}^{k-1} m(z^i_{t+j}, z^i_{t+j+1}; \theta)\}(R^i_k - X)^+\tau. \tag{15}$$

Note that the first state variable, the level, is the 3-month LIBOR.

VI. Empirical Results

In this section, we present and compare out-of-sample pricing results using a series of different models. In particular, we compare the pricing performance of 1. a Gaussian VAR(1) model, 2. a Gaussian VAR-GARCH(1,1) model, 3. an unconditional Non-Gaussian VAR-GARCH(1,1) model and 4. a conditional Non-Gaussian VAR-GARCH(1,1) model. For the last three models, we also report the pricing errors using linear, quadratic, and cubic pricing kernels. The pricing errors we obtain are of similar size to those in Jaganathan, Kaplin, and Sun (2003), where they use caps data to evaluate multi-factor CIR models.

A. Gaussian VAR(1) Factor Dynamics

Table 4 reports the pricing errors from a Gaussian VAR(1) model for the factor dynamics with linear log pricing kernel.[9] This is basically a discrete time version of the two factor Vasicek model, where the state variable z follow a first-order autoregression on its own lag, with correlated normal innovations. The conditional variance-covariance matrix of the factors is therefore constant. It is clear from Table 4 that the Gaussian VAR(1) model is inadequate for pricing caps. The Gaussian VAR(1) model does not capture the volatility dynamics properly. As a result, it cannot generate enough variability in the short rate. Therefore, most caplets end up out of the money.

B. Gaussian VAR-GARCH(1,1) Factor Dynamics

Table 5 presents the pricing errors from a Gaussian VAR-GARCH(1,1) model for the LIBOR dynamics with linear, quadratic, and cubic log pricing kernels, respectively. Comparing with those from the Gaussian VAR(1) model in last section, the pricing errors from the Gaussian VAR-GARCH(1,1) model are significantly lower. This indicates that conditional heteroskedasticity in the factors is an important feature that a model must capture to be successful in pricing interest rate caps.

One year caps have an average absolute percentage errors of 137% (18.6 basis points in mean absolute pricing error), much greater than those for longer maturity caps, which are between 52% and 72%. This makes sense since the one year cap has fewer caplets and the

[9] To save space, we do not report the results from using non-linear log pricing kernels for the Gaussian VAR(1) model. The results are similar to the linear log pricing kernel case.

caplets are all short maturity ones. Therefore, it is more sensitive to any mis-specifications in the volatility dynamics. If volatility is mean-reverting, for long maturity caps, the impacts of mis-specifications in the volatility dynamics can be eventually "washed out". In the short run, however, the volatility dynamics is crucial in determining the distribution of the short rate.

The corresponding pricing errors are basically the same across linear, quadratic, and cubic log pricing kernels. This suggests that increasing the order of the log pricing kernels do not improve cap pricing, perhaps because the underlying factor dynamics are still mis-specified.

C. Unconditional Non-Gaussian VAR-GARCH(1,1) Factor Dynamics

In Table 6, where the factor dynamics are estimated using a Non-Gaussian model with a Gaussian VAR-GARCH(1,1) leading term. This leading term captures the conditional heteroskedasticity in the data. Any non-normality in innovations are described by a fourth-order Hermite polynomial transformation of the conditional normal distribution.

The pricing errors across linear, quadratic, and cubic log pricing kernels are only slightly different. The average absolute pricing error for one year caps is 8.55 basis points for the linear log pricing kernel model and 8.60 for the cubic log pricing kernel model, while the average absolute pricing error for ten year caps is 358.0 basis points for the linear log pricing kernel model and 344.7 basis points for the cubic log pricing kernels. It seems that the linear log pricing kernel prices short-maturity caps slightly better than the cubic log pricing kernel, while the cubic log pricing kernel is slightly better in pricing the long-maturity caps. Again, similarly to what we observe in section B, increasing the order of the log pricing kernel yields only little in the ability of the model to price caps.

Comparing Table 6 with Table 5, we notice that pricing errors for short maturity caps are reduced significantly. The average absolute pricing error for one-year caps are reduced from 18.6 basis points to 8.6 basis points. Average relative absolute pricing error are reduced from 137.0% to 57.8%. The average relative absolute pricing error for other short-maturity caps, the two-year, three-year, and four-year caps are also reduced from 71.4%, 57.8%, and 53.5% to 46.6%, 46.4%, and 48.5% respectively. (Without loss of generality, all of the above are based on the linear log pricing kernel case.) The results show that non-normality play an important role in pricing short maturity caps. In addition, it confirms our observation from last section that short maturity caps are more sensitive to the volatility dynamics.

D. Conditional Non-Gaussian VAR-GARCH(1,1) Factor Dynamics

Table 7 reports the pricing errors from a model very similar to the unconditional Non-Gaussian VAR-GARCH(1,1) model, the only difference being that the parameters in the Hermite polynomial are allowed to be linear functions of the factors, i.e. the level and the slope. The conditional distribution is conditional on the factors, hence the name of the model. The volatility dynamics of the factors in this model is flexible enough to allow the conditional non-normal distribution to be a time-varying function of the level and the slope of the yield curve.

This model best describes the factor dynamics among the ones we have examined. Table 7 shows pricing errors are reduced as we move from linear log pricing kernels to higher order log pricing kernels, such as quadratic and cubic log pricing kernels, though the improvement from quadratic log pricing kernel to cubic log pricing kernel is relatively small. This is particularly pronounced for long-term caps. For example, the average absolute pricing error are reduced from 292 basis points for the linear kernel case to 249 and 245 basis points for quadratic and cubic kernel case respectively. That is a reduction of 50 basis points in pricing error. This result shows that once we have a model that closely captures the time varying non-normality in the factor dynamics, non-linear log pricing kernels price caps significantly better than linear log pricing kernels. This is especially pronounced for long term caps.

Comparing Table 7 with Table 6, we find that cap pricing errors are reduced when we allow conditional density to be time-varying polynomial transformation of a normal density for the factor dynamics. The reduction is most evident for long term caps. For ten-year caps, the average absolute pricing errors are reduced from 358, 361, and 345 basis points to 291, 249, and 245 basis points for linear, quadratic and cubic log pricing kernels respectively.

VII. Conclusion

In this paper we investigate the volatility dynamics of the short term interest and nonlinear pricing kernels. We explore their implications on out-of-sample pricing of interest caps. In particular, we compare the pricing results of 1. a Gaussian VAR(1) model, 2. a Gaussian VAR-GARCH(1,1) model, 3. an unconditional Non-Gaussian VAR-GARCH(1,1) model and 4. a conditional Non-Gaussian VAR-GARCH(1,1) model. We also examine the impact of using linear, quadratic, and cubic log pricing kernels on pricing interest caps. We find:

1. Conditional Heteroskedasticity plays an important role in the pricing of interest caps. Ignoring this fact, such as the case of two-factor Vasicek model, the short rate does not exhibits the variability needed for the options to be in the money.

2. If the underlying factor dynamics are mis-specified, increasing the order of log pricing kernel does not improve pricing performance.

3. Conditional non-normality is also an important feature in pricing interest caps. It is particularly crucial for short maturity caps.

4. In addition to non-normality, the conditional density of the short rate appears to be time varying, depending on the level and the slope of term structure. Incorporating this feature in the model significantly reduces the pricing errors for interest rate caps. The effect is especially pronounced for long maturity caps.

5. Once we have found a model that closely captures the time-series characteristics of the short rate, i.e. time-varying conditional non-normality, increasing the order of the logarithm of the pricing kernel by moving from linear function to quadratic or cubic functions further improves pricing performance. This presents a challenge to affine type models, in which the logarithm of pricing kernel is an affine function of state variables.

The results from this paper open up several avenues for future research. First, this paper adds evidence to the literature about non-linearity in the diffusion term of the short rate process. In particular, we find that the diffusion term is affected by at least two factors, i.e. the level and the slope of the term structure. This agrees with the findings by Boudoukh, Richardson, Stanton, and Whitelaw (1999). What parametric functional forms is then appropriate for describing this relationship? Linear, quadratic or some other functional forms? Second, we find that with a model that closely captures the time-series property of factor-dynamics of the short rate, quadratic and cubic log-pricing kernels outperform log-linear pricing kernels. What type of parametric models imply quadratic or even cubic log-pricing kernels? Finally, even with the most flexible model, the conditional Non-Gaussian VAR-GARCH(1,1) model, there remain significant pricing errors for caps. This study therefore confirms, from a nonparametric perspective, recent research of diffusion-only type parametric models that fit libor and swap rates but do not price derivatives well. The success of jump-type models such as that used by Jarrow, Li, and Zhao (JF, forthcoming) seems to suggest that jumps are needed to explain the prices of libor options.

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Table 1

Descriptive Statistics for U.S. Dollar LIBOR and Swap Rates (in percentages)

	3ML	6ML	1Y	2Y	3Y	4Y	5Y	7Y	10Y
mean	5.95	6.05	6.34	6.74	7.00	7.20	7.34	7.56	7.75
standard deviation	1.78	1.77	1.75	1.68	1.60	1.55	1.51	1.49	1.44

Table 2

Descriptive Statistics for Weekly Changes to U.S. Dollar LIBOR and Swap Rates (in Basis Points)

	3ML	6ML	1Y	2Y	3Y	4Y	5Y	7Y	10Y
mean	-0.217	-0.206	-0.182	-0.177	-0.192	-0.208	-0.225	-0.255	-0.28
standard deviation	0.139	0.142	0.174	0.170	0.165	0.162	0.159	0.156	0.153

Table 3

Black's Volatility for Interest Caps
(in Percentages)

	1Y	2Y	3Y	4Y	5Y	7Y	10Y
mean	11.9	14.9	16.0	16.2	16.2	15.9	15.4
standard deviation	3.2	3.1	2.7	2.5	2.3	1.9	1.5

Table 4

Pricing Errors of the Gaussian VAR(1) Model

	1Y	2Y	3Y	4Y	5Y	7Y	10Y
A. Linear Log Pricing Kernel							
Mean Pricing Error(b.p.)	-13.8	-53.7	-105.3	-162.7	-223.4	-347.8	-533.9
Mean Absolute Pricing Error(b.p.)	13.8	53.7	105.3	162.7	223.4	347.8	533.9
% Absolute Pricing Error(%)	97.5	99.5	99.8	99.9	99.9	100.0	100.0
Maximum Pricing Error(b.p.)	32.6	80.3	144.1	213.5	287.0	429.7	628.8

Table 5

Pricing Errors of the Gaussian VAR-GARCH(1,1) Model

	1Y	2Y	3Y	4Y	5Y	7Y	10Y
A. Linear Log Pricing Kernel							
Mean Pricing Error(b.p.)	8.7	-2.1	-25.2	-55.6	-91.6	-176.3	-323.8
Mean Absolute Pricing Error(b.p.)	18.6	40.1	63.4	89.7	118.9	188.8	326.9
% Absolute Pricing Error(%)	137.0	71.4	57.8	53.5	52.2	53.6	60.5
Maximum Pricing Error(b.p.)	75.9	160.7	193.3	192.6	255.3	391.6	588.8
B. Quadratic Log Pricing Kernel							
Mean Pricing Error(b.p.)	8.7	-2.0	-25.1	-55.7	-92.2	-177.8	-326.7
Mean Absolute Pricing Error(b.p.)	18.7	40.3	63.8	90.4	120.0	190.6	329.9
% Absolute Pricing Error(%)	137.2	71.7	58.2	53.9	52.7	54.1	61.1
Maximum Pricing Error(b.p.)	76.1	161.8	195.3	195.8	256.9	394.2	593.0
C. Cubic Log Pricing Kernel							
Mean Pricing Error(b.p.)	8.7	-2.0	-25.2	-55.9	-92.6	-178.5	-327.9
Mean Absolute Pricing Error(b.p.)	18.7	40.3	63.9	90.4	120.2	191.0	331.0
% Absolute Pricing Error(%)	137.3	71.7	58.3	54.0	52.8	54.2	61.3
Maximum Pricing Error(b.p.)	76.2	162.2	195.8	195.2	257.1	394.6	593.5

Table 6

Pricing Errors of the Unconditional Non-Gaussian VAR-GARCH(1,1) Model

	1Y	2Y	3Y	4Y	5Y	7Y	10Y
A. Linear Log Pricing Kernel							
Mean Pricing Error(b.p.)	-4.5	-20.8	-47.6	-81.2	-120.6	-208.8	-358.0
Mean Absolute Pricing Error(b.p.)	8.6	26.9	51.6	82.6	120.6	208.8	358.0
% Absolute Pricing Error(%)	57.8	46.6	46.4	48.5	52.1	58.5	66.0
Maximum Pricing Error(b.p.)	28.1	64.5	121.8	181.8	247.6	377.2	559.9
B. Quadratic Log Pricing Kernel							
Mean Pricing Error(b.p.)	-4.6	-21.1	-48.3	-82.5	-122.4	-211.2	-360.2
Mean Absolute Pricing Error(b.p.)	8.6	27.5	53.2	85.2	123.6	212.0	360.5
% Absolute Pricing Error(%)	58.4	47.9	48.1	50.4	53.7	59.6	66.6
Maximum Pricing Error(b.p.)	28.1	63.2	121.1	173.1	234.2	361.1	546.3
C. Cubic Log Pricing Kernel							
Mean Pricing Error(b.p.)	-4.5	-20.7	-47.1	-79.8	-118.0	-202.6	-344.3
Mean Absolute Pricing Error(b.p.)	8.6	27.3	52.1	82.6	119.3	203.4	344.7
% Absolute Pricing Error(%)	58.2	47.4	47.0	48.7	51.7	57.1	63.6
Maximum Pricing Error(b.p.)	28.1	64.2	119.9	181.5	247.6	378.0	557.8

Table 7

Pricing Errors of the Conditional Non-Gaussian VAR-GARCH(1,1) Model

	1Y	2Y	3Y	4Y	5Y	7Y	10Y
A. Linear Log Pricing Kernel							
Mean Pricing Error(b.p.)	-8.0	-34.6	-69.8	-107.1	-142.6	-199.3	-216.9
Mean Absolute Pricing Error(b.p.)	10.0	36.5	70.1	107.1	142.8	201.7	291.4
% Absolute Pricing Error(%)	67.1	65.7	64.9	64.3	62.5	56.5	53.6
Maximum Pricing Error(b.p.)	31.2	69.3	130.1	186.7	251.3	375.2	734.5
B. Quadratic Log Pricing Kernel							
Mean Pricing Error(b.p.)	-7.9	-34.3	-69.0	-104.6	-136.7	-186.6	-199.2
Mean Absolute Pricing Error(b.p.)	9.9	36.4	69.5	104.6	136.7	186.7	248.9
% Absolute Pricing Error(%)	66.8	65.6	64.5	62.9	60.0	52.6	45.9
Maximum Pricing Error(b.p.)	23.6	68.8	127.2	190.0	257.2	383.6	623.3
C. Cubic Log Pricing Kernel							
Mean Pricing Error(b.p.)	-7.9	-34.3	-69.2	-104.6	-137.6	-189.3	-207.4
Mean Absolute Pricing Error(b.p.)	10.0	36.4	69.7	104.7	137.6	190.6	244.7
% Absolute Pricing Error(%)	67.4	65.5	64.6	62.9	60.2	53.4	45.2
Maximum Pricing Error(b.p.)	31.2	69.2	126.7	188.7	254.9	379.5	499.0

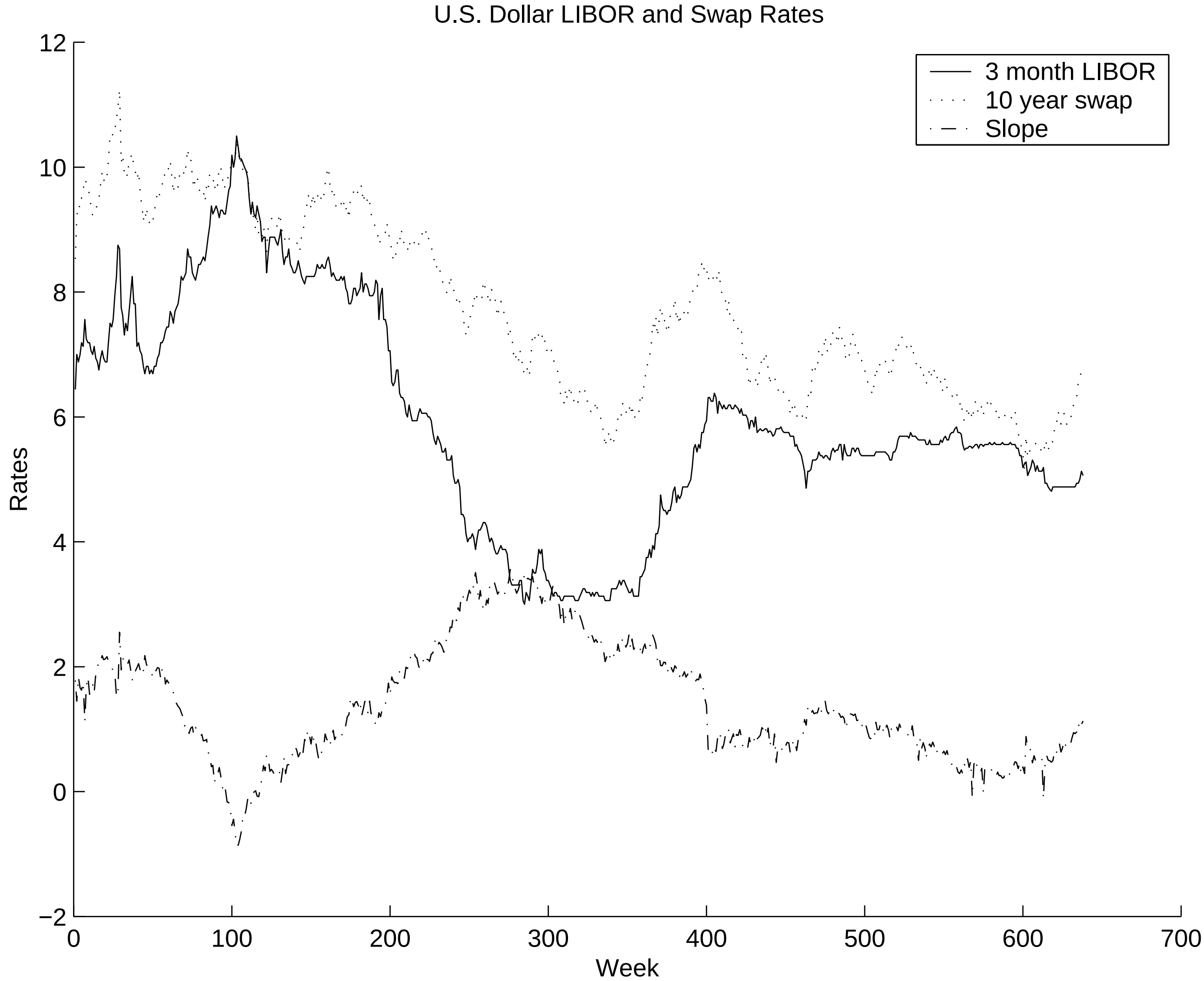

Figure 1. U.S. Dollar LIBOR and Swap Rates.

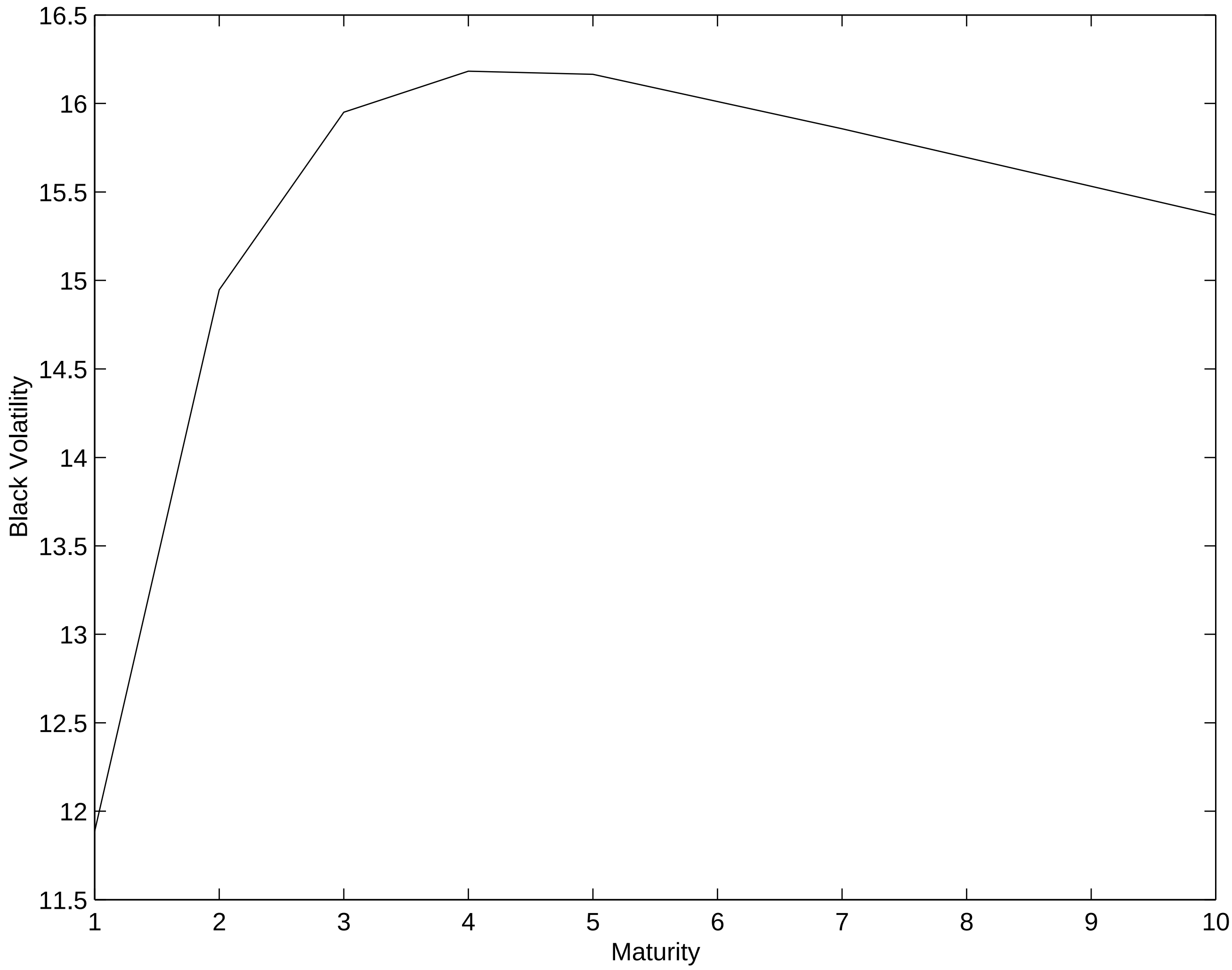

Figure 2. Average Cap Black's Volatility.

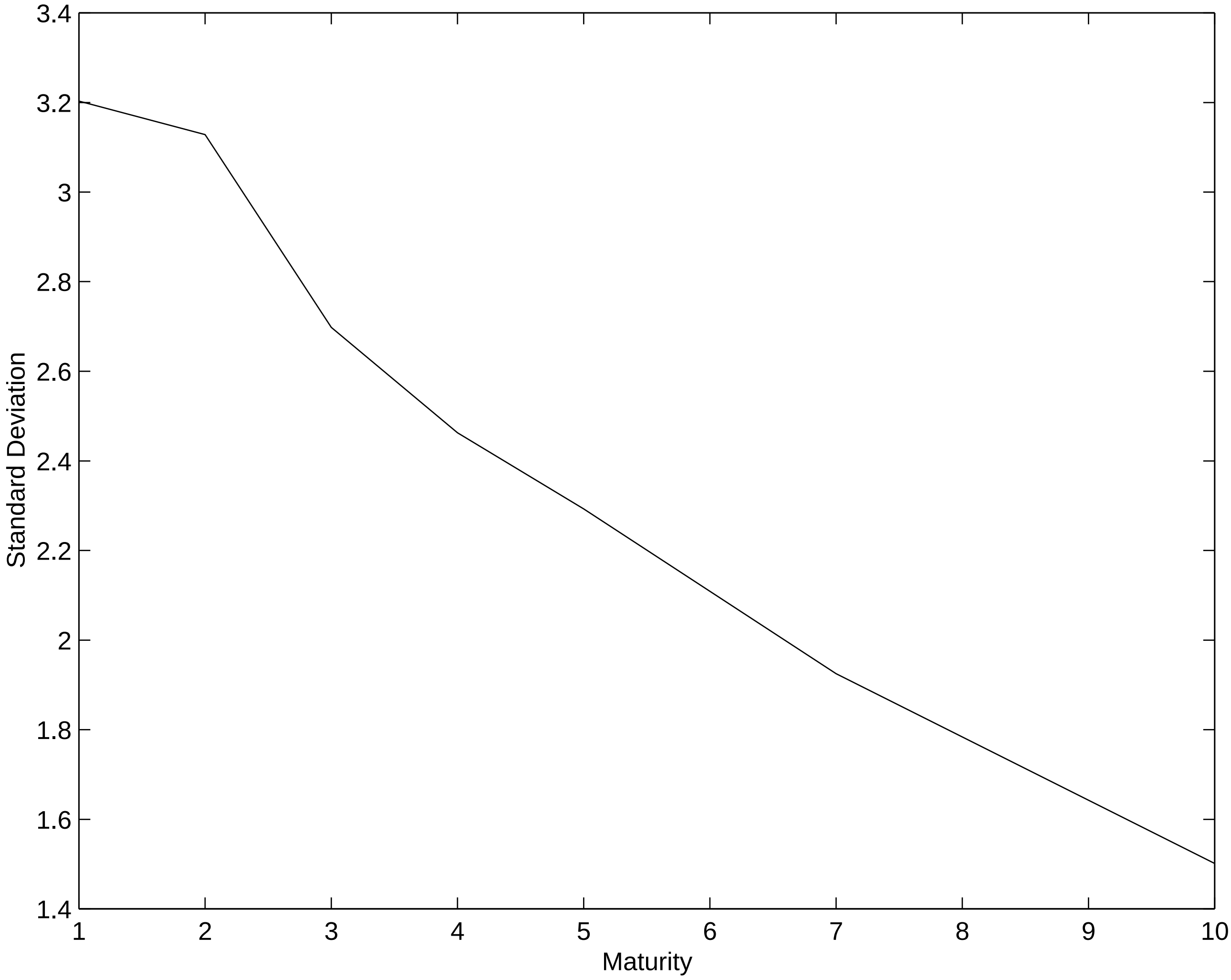

Figure 3. Standard Deviations of Cap Black's Volatility.